n1

ex1 $\quad f(x) = \dfrac{x^4 + x^3 - x^2 - x}{1 - x}$

a) $D_f$ : pb si $1-x=0 \Leftrightarrow x=1$ $\qquad D_f = \mathbb{R} \setminus \{1\}$

$Z_f$ : $x^4+x^3-x^2-x=0$

$\Leftrightarrow x^2(x+1) - x(x+1) = 0$

$\Leftrightarrow x(x+1)(x^2-1) = 0$

$\Leftrightarrow x(x+1)(x-1)(x+1) = 0$

$x=0$ ou $x=-1$ ou $x=1$ $\qquad Z_f = \{-1;0;1\}$

b) $f(1{,}9) \simeq -15{,}98$ $\qquad f(2{,}1) \simeq -20{,}18$ $\qquad$ (cf "table" de la calculatrice ou geogebra)

$f(1{,}99) \simeq -17{,}8$ $\qquad f(2{,}01) \simeq -18{,}21$

$f(1{,}999) \simeq -17{,}98$ $\qquad f(2{,}001) \simeq -18{,}02$

c) Conjecture : $\lim\limits_{x \to 2} f(x) = -18$

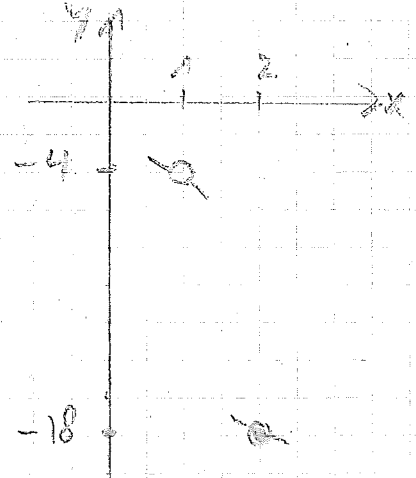
d) $f(0{,}9) \simeq -3{,}25$ $\qquad f(1{,}1) \simeq -4{,}85$

$f(0{,}99) \simeq -3{,}92$ $\qquad f(1{,}01) \simeq -4{,}08$

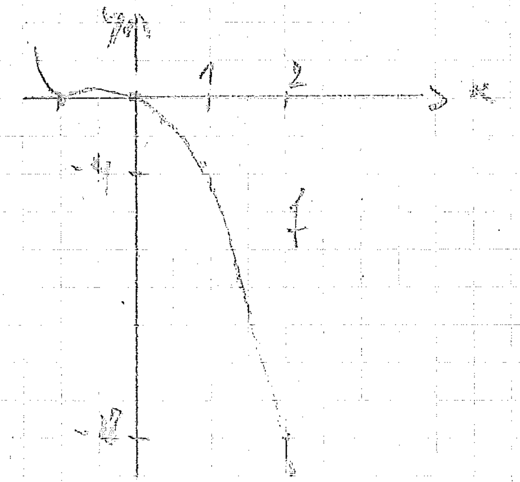
$f(0{,}999) \simeq -3{,}99$ $\qquad f(1{,}001) \simeq -4{,}01$

e) Conjecture : $\lim\limits_{x \to 1} f(x) = -4$

f)




Mentalement




avec Geogebra

ne montre pas le pb à $x=1$ !!!

## ex 2

a) 3

b) 3

c) 1

d) 4

e) 0

f) 2

g) 1

h) $-\infty$

i) $\nexists$

j) 1

k) 0

l) 4

m) 3

n) 4

## ex 3

a) $\lim\limits_{x\to 1} f(x) \overset{PrL6}{=} \dfrac{\lim\limits_{x\to 1} x^2-6x+8}{\lim\limits_{x\to 1} x^2+5x+4} \overset{PrL5}{=} \dfrac{1^2-6\cdot 1+8}{1^2+5\cdot 1+4} = \dfrac{3}{10}$

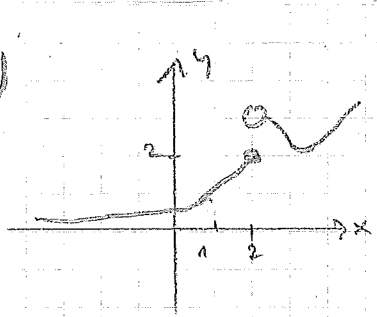
b) $\lim\limits_{x\to 0} f(x) \overset{PrL4}{=} \dfrac{1}{3}\lim\limits_{x\to 0}\sqrt{x^2-4x+16} \overset{PrL7}{=} \dfrac{1}{3}\sqrt{\lim\limits_{x\to 0} x^2-4x+16}$

$\overset{PrL5}{=} \dfrac{1}{3}\sqrt{0^2-4\cdot 1+16} = \dfrac{1}{3}\sqrt{12}$

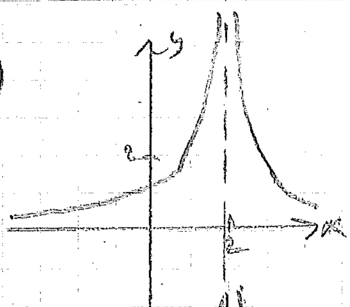
c) $\lim\limits_{x\to -\frac{\pi}{2}} f(x) = \cos\left(\lim\limits_{x\to -\frac{\pi}{2}} 2x\right) = \cos\left(2\cdot\left(-\frac{\pi}{2}\right)\right) = \cos(-\pi) = -1$

## ex 5

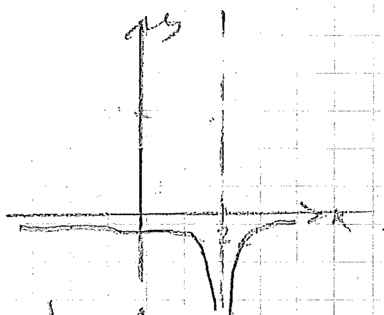
i)



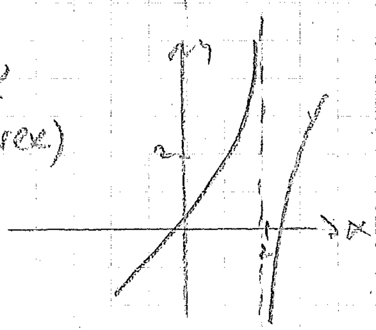
ii)

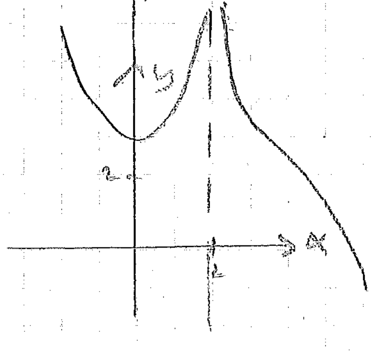


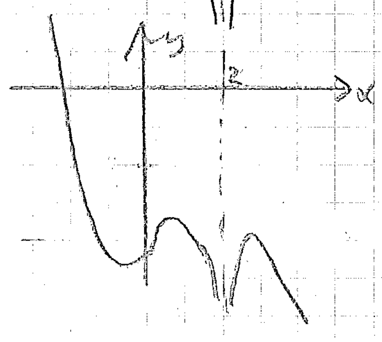
iii)



ou (par ex.)







ex 6

i) $f_1(x) = \frac{1}{x-2}$ $f_2(x) = \frac{2}{x-2}$

ii) $f_1(x) = \frac{1}{(x-2)^2}$ $f_2(x) = \frac{2}{(x-2)^2}$

iii) $f_1(x) = -\frac{1}{(x-2)^2}$ $f_2(x) = -\frac{2}{(x-2)^2}$

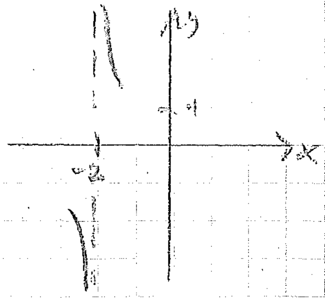
ex 4

a) $\lim\limits_{x\to -2} f(x) = \text{"}\frac{2}{0}\text{"}$ : type $\frac{1}{0}$

$\lim\limits_{x\to -2^+} f(x) = \lim\limits_{x\to -2^+} \frac{\cancel{2}\,1}{\cancel{2}(x+2)} = \frac{1}{0^+} = +\infty$

$\lim\limits_{x\to -2^-} f(x) = \frac{1}{0^-} = -\infty$

$\Bigg\}$ donc $\lim\limits_{x\to -2} f(x) \nexists$

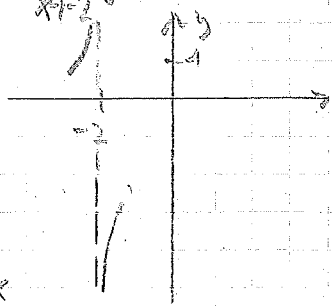



b) $\lim\limits_{x\to -2} \frac{-2}{2x+4} = \text{"}\frac{-2}{0}\text{"}$ : type $\frac{1}{0}$

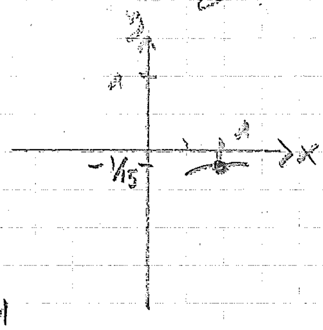
$\lim\limits_{x\to -2^+} f(x) = \lim\limits_{x\to -2^+} \frac{-\cancel{2}\,1}{\cancel{2}(x+2)} = \frac{-1}{0^+} = -\infty$

$\lim\limits_{x\to -2^-} f(x) = \frac{-1}{0^-} = +\infty$

$\Bigg\}$ donc $\lim\limits_{x\to -2} f(x) \nexists$




c) $\lim\limits_{x\to 1} f(x) = \frac{2-1}{1-16} = \frac{1}{-15}$

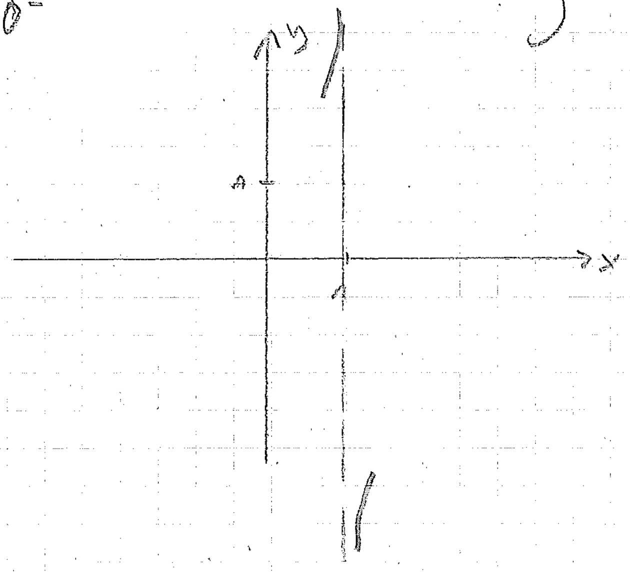



d) $\lim\limits_{x\to 1} f(x) = \text{"}\frac{1}{0}\text{"}$ : type $\frac{1}{0}$

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} \frac{x-2}{(x+4)(x-1)} = \frac{-1}{5\cdot 0^+} = -\infty$

$\lim\limits_{x\to 1^-} f(x) = \frac{-1}{5\cdot 0^-} = +\infty$

$\Bigg\}$ $\lim\limits_{x\to 1} f(x) \nexists$



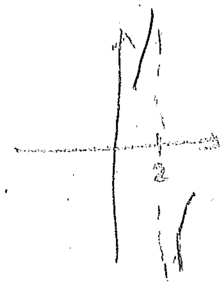
ex 4 (suite)

e) $\lim\limits_{x\to 2} \frac{3x-7}{x-2}$ type "$\frac{1}{0}$"

$\lim\limits_{x\to 2^-} f(x) = \frac{3\cdot 2-7}{2^- -2} = \frac{-1}{0^-} = +\infty$

$\lim\limits_{x\to 2^+} f(x) = \frac{3\cdot 2-7}{2^+ -2} = \frac{-1}{0^+} = -\infty$

$\Big\}$ $\lim\limits_{x\to 2} f(x) \nexists$

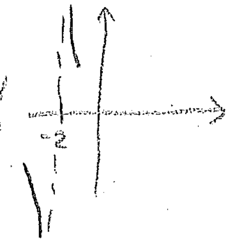



f) $\lim\limits_{x\to -2} \frac{-3x+2}{x+2}$ type "$\frac{1}{0}$"

$\lim\limits_{x\to -2^-} f(x) = \lim\limits_{x\to -2^-} \frac{-3(-2)+2}{-2^- +2} = \frac{8}{0^-} = -\infty$

$\lim\limits_{x\to -2^+} f(x) = \lim\limits_{x\to -2^+} \frac{-3(-2)+2}{-2^+ +2} = \frac{8}{0^+} = +\infty$

$\Big\}$ $\lim\limits_{x\to -2} f(x) \nexists$

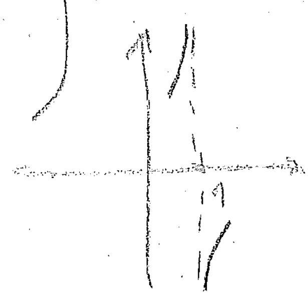



g) $\lim\limits_{x\to 1} \frac{3x}{1-x^2}$ type "$\frac{1}{0}$"

$\lim\limits_{x\to 1^+} f(x) = \underbrace{\lim\limits_{x\to 1^+} \frac{3x}{(1-x)(1+x)}}_{\text{⚠ on factorise le plus possible le dénominateur pour contrôler le signe}} = \frac{3\cdot 1}{0^-\cdot 2} = -\infty$

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} \frac{3x}{(1-x)(1+x)} = \frac{3}{0^+\cdot 2} = +\infty$

$\Big\}$ $\lim\limits_{x\to 1} f(x) \nexists$

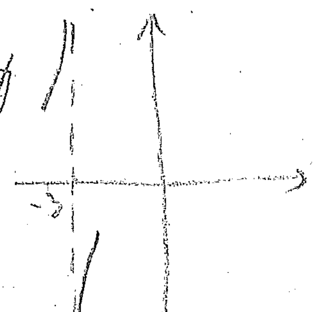



h) $\lim\limits_{x\to -3} \frac{5x}{(x+3)^3}$ type "$\frac{1}{0}$"

$\lim\limits_{x\to -3^+} f(x) = \frac{5(-3)}{(0^+)^3} = \frac{-15}{0^+} = -\infty$

$\lim\limits_{x\to -3^-} f(x) = \frac{5(-3)}{(0^-)^3} = \frac{-15}{0^-} = +\infty$

$\Big\}$ $\lim\limits_{x\to -3} f(x) \nexists$




i) $\lim\limits_{x\to -3} \frac{5x}{(x+3)^2}$ type "$\frac{1}{0}$"

$\lim\limits_{x\to -3^+} f(x) = \lim\limits_{x\to -3^+} \frac{5(-3)}{(0^+)^2} = -\infty$

$\lim\limits_{x\to -3^-} f(x) = \lim\limits_{x\to -3^-} \frac{5(-3)}{(0^-)^2} = -\infty$

$\Big\}$ $\lim\limits_{x\to -3} f(x) = -\infty$