Complétion du carré – cas général

$$
\begin{aligned}
ax^2+bx+c &= a\left(x^2+\frac{b}{a}x\right)+c \\
&= a\left(x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2\right)+c \\
&= a\left(\left(x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2\right)-\left(\frac{b}{2a}\right)^2\right)+c \\
&= a\left(\left(x^2+\frac{b}{a}x+\left(\frac{b^2}{4a^2}\right)\right)-\frac{b^2}{4a^2}\right)+c \\
&= a\left(\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}\right)+c \\
&= a\left(x+\frac{b}{2a}\right)^2-\frac{ab^2}{4a^2}+c \\
&= a\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a}+c \\
&= a\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a}+\frac{4ac}{4a} \\
&= a\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4ac}{4a}
\end{aligned}
$$